# Measuring the Similarity of Nodes in Signed Social Networks with Positive and Negative Links

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**Abstract.** Similarity measure in non-signed social networks has been extensively studied for decades. However, how to measure the similarity of two nodes in signed social networks remains an open problem. It is challenging to incorporate both positive and negative relationships simultaneously in signed social networks due to the opposite opinions implied by them. In this paper, we study the similarity measure problem in signed social networks. We propose a basic node similarity measure that can utilize both positive and negative relations in signed social networks by comparing the immediate neighbors of two objects. Moreover, we exploit the propagation of similarity in networks. Finally, we perform extensive experimental comparison of the proposed method against existing algorithms on real data set. Our experimental results show that our method outperforms other approaches.

Keywords: similarity measure, signed networks, positive and negative links

# 1 Introduction

Measuring the similarity of nodes in signed social networks[1, 2, 3] is an important but still not fully explored problem due to the following challenges. First, the existence of negative links in signed networks challenges many concepts for unsigned networks. In addition, it is challenging to incorporate both positive and negative relationships simultaneously in signed networks. As shown in Fig. 1, users can express trust or distrust on others in Epinions.com, in which "+" indicates trust and "-" indicates distrust.

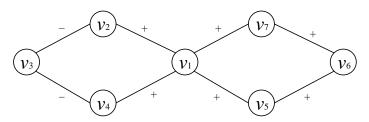


Fig. 1. Example of Signed Social Network.

adfa, p. 1, 2011. © Springer-Verlag Berlin Heidelberg 2011 We can see from Fig.1, user  $v_1$  and user  $v_3$  have two common neighbors, user  $v_1$  and user  $v_6$  have two common neighbors as well. If we ignore the polarity of this social network, we can conclude that user  $v_3$  and user  $v_6$  are both similar to user  $v_1$ . However, when we take into account the polarities of the links, things go differently. From the Fig.1, we can see that user  $v_1$  and user  $v_3$  have different evaluations for the same persons, while user  $v_1$  and user  $v_6$  have identical evaluations for the same persons. So we can deduce that user  $v_1$  is more similar to user  $v_6$ , and user  $v_1$  is more dissimilar to user  $v_3$ .

In this paper, we study the problem of measuring the similarity of nodes in signed social networks. Compared to existing approaches, our method considers the influence of negative relationships in addition to positive relationships. In particular, we define a basic similarity measure that captures effectively the proximity of two nodes in signed social networks. Besides, we also exploit the propagation of similarity in social networks. For example, considering the user  $v_3$  and user  $v_6$  in Fig. 1, these two users have no immediate neighbors in the signed social network, however, we can infer that user  $v_3$  is dissimilar to user  $v_6$  because user  $v_3$  is dissimilar to user  $v_1$  and user  $v_1$  is similar to user  $v_6$ .

The major contributions of this paper are summarized as below.

- It investigates the similarity measure problem in signed social networks, a new but increasingly important issue due to the continuous development of social networks.
- It proposes a basic similarity measure method in signed social networks, which can utilize both positive and negative relationships.

The rest of this paper is organized as follows. Section 2 summarizes the related work, whereas section 3 denotes a node similarity measure in signed social networks. Experimental results are given in section 4. Finally, section 5 concludes this paper.

### 2 Related Work

Unsigned social networks have been studied for decades [4, 5, 6]. There are many similarity measure method based on unsigned social networks. Traditional approaches to measure the similarity of nodes are mainly based on their feature values, such as Jaccard coefficient, Cosine similarity and Euclidean distance. However, these similarity measures are mainly focus on the nodes features, they do not consider link structures among objects.

The link based approaches measure the similarity of nodes defined on networks, such as shortest path algorithm, RWR (Random Walk with Restart) [7], SimRank [8], Personalized PageRank [9] and etc. Liben and Kleinberg [10] claimed that the identification of the shortest path between any pair of nodes in a graph can be used for friend recommendation. RWR iteratively explores the global structure of the network to estimate the proximity between two nodes. Starting at a node, the walker faces two choices at each step: either moving to a randomly chosen neighbor, or jumping back to the starting node. SimRank is a symmetric similarity measure that says "two objects are considered to be similar if they are referenced by similar objects". Personalized PageRank is an asymmetrical similarity measure that evaluates the probability starting from source nodes to target nodes by RWR. However, all above these methods do not consider the negative links, i.e., these studies are built on a fundamental assumption that all links in networks are positive.

FriendTNS [11, 12] discussed the similarity measure problem in signed social networks based on status theory. FriendTNS evaluates the similarity of two nodes in terms of their positive in-degree, negative out-degree, positive out-degree and negative indegree. However, FriendTNS considers the degree of nodes only, it does not consider the relationship between two nodes. Besides, FriendTNS cannot apply in undirected signed social networks.

# **3** Node Similarity Measure

In this section, we define a basic node similarity measure to determine the proximity between a pair of nodes in signed social networks. For node  $v_i$  and node  $v_j$ , we define a specific function  $sim(v_i, v_j)$  to express their corresponding similarity.

### 3.1 Basic Idea

To capture proximity between two nodes, we consider the example shown in Fig. 2. Fig. 2(a) means user  $u_1$  and user  $u_2$  have many common friends, and Fig. 2(b) means that user  $u_1$  and user  $u_2$  have many common enemies. We can infer that  $u_1$  and  $u_2$  are likely to be similar, because they have the same evaluations on the same people. In contrast, Fig. 2(c) and Fig. 2(d) show that the evaluations of user  $u_1$  and user  $u_2$  on the same people are diametrically opposite. So, we can infer that  $u_1$  and  $u_2$  are most likely not similar.

This inspires us that if a pair of users will have high similarity in a signed social network, they should be satisfied the following condition:

- They have many common friends.
- They have many common enemies.

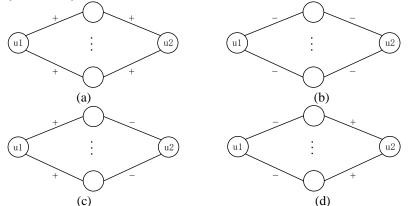


Fig. 2. Example of two neighbor connected users (nodes) in a signed social network.

#### 3.2 Similarity Measure in Undirected Signed Social Networks

Based on the above intuition, in an undirected signed social network, given a pair of nodes  $v_i$  and  $v_j$ , we can determine their similarity by considering their immediate neighbor sets. The followings are the immediate neighbor nodes, positive immediate neighbor nodes and negative immediate neighbor nodes of node  $v_i$ .

$$N_{i} = \{v_{k} \mid (v_{i}, v_{k}) \in \mathcal{E}\} + \{v_{i}\}$$
(1)

$$N_i^+ = \{v_k \mid (v_i, v_k) \in \mathcal{E}^+\} + \{v_i\}$$
(2)

$$N_i^- = \{ v_k \mid (v_i, v_k) \in \mathcal{E}^- \}$$
(3)

We have discussed above that the more same evaluations on common neighbors two nodes have, the more similar they are. In contrast, the more contrary evaluations on common neighbors two nodes have, the more dissimilar they are.

Given a pair of nodes  $v_i$  and  $v_j$ , we can get the sets of nodes that they hold same evaluations and different evaluations.

$$C_s(v_i, v_j) = \{v_k \mid \left(v_k \in N_i^+ \land v_k \in N_j^+\right) \lor \left(v_k \in N_i^- \land v_k \in N_j^-\right)\}$$
(4)

$$C_d(v_i, v_j) = \{ v_k \mid (v_k \in N_i^+ \land v_k \in N_j^-) \lor (v_k \in N_i^- \land v_k \in N_j^+) \}$$
(5)

The similarity of these two nodes can be determined as follows.

$$sim(v_i, v_j) = \frac{|c_s(v_i, v_j)| - |c_d(v_i, v_j)|}{|v_i \cup v_j|}$$
(6)

The node similarity measure in undirected signed social networks returns values into the interval [-1, 1]. Note that the maximum value of similarity (equal 1) and minimum value of similarity (equal -1) can be reached. When the two nodes have totally the same neighbors and the evaluation on each neighbor is the same, then the similarity of these two nodes is equal 1. However, when the two nodes have totally the same neighbors, but the evaluation on each neighbor is completely different, then the similarity of these two nodes is equal to -1. The larger the value, the more similar these two nodes are. The similarity value 1 indicates that the two nodes are extremely similar, and the value -1 indicates that the two nodes are extremely dissimilar.

Now, let us calculate some similarity values on the graph of Fig. 1 using Eq. 6. Considering the node  $v_1$  and node  $v_3$ ,  $N_1 = \{v_1, v_2, v_4, v_5, v_7\}$ ,  $N_3 = \{v_2, v_3, v_4\}$ ,  $N_1^+ = N_1$ ,  $N_1^- = \emptyset$ ,  $N_3^+ = \{v_3\}$ ,  $N_3^- = \{v_2, v_4\}$ ,  $C_s(v_1, v_3) = \emptyset$ ,  $C_d(v_1, v_3) = \{v_2, v_4\}$ , so  $sim(v_1, v_3) = \frac{0-2}{6} = -\frac{1}{3}$ . Similarly, the similarity between node  $v_1$  and  $v_6$  is:  $sim(v_1, v_6) = \frac{1}{3}$ . Thus, the similarity score between nodes  $v_1$ ,  $v_3$  is less than that of  $v_1$ ,  $v_6$ , and the result is also consistent with our intuitive experience.

#### 3.3 Similarity Measure in Directed Signed Social Networks

In this section, we present how to measure similarity of a pair of nodes in a directed signed social network. In a directed social network, a user can express his attitudes to

others, but can also receive the evaluations by other people. So, we separate neighbor set into four parts: (1) positive input neighbor set; (2) negative input neighbor set; (3) positive output neighbor set; (4) negative output neighbor set. Taking Fig. 3 as an example, we consider the similarity between node  $u_1$  and node  $u_2$ .

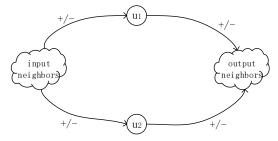


Fig. 3. Illustration of the intuition that measure similarity in a directed signed social network.

Based on the above intuition,  $u_1$  and  $u_2$  may be similar if:

- The evaluations on the two users that given by a lot of people are the same.
- The evaluations on a group of persons that given by the two users are the same.

Besides, if the two users hold different evaluations on many same persons and many people have different opinions about the two users, they may be dissimilar. It is an extension of the intuition we described in section 3.1.

The followings are the positive input neighbor set, negative input neighbor set, positive output neighbor set and negative output neighbor set of node  $v_i$  in a directed signed social network.

$$I_i^+ = \{ v_k \mid (v_k, v_i) \in \mathcal{E}^+ \} + \{ v_i \}$$
(7)

$$I_{i}^{-} = \{ v_{k} \mid (v_{k}, v_{i}) \in \mathcal{E}^{-} \}$$
(8)

$$O_i^+ = \{ v_k \mid (v_i, v_k) \in \mathcal{E}^+ \}$$
(9)

$$O_i^- = \{ v_k \mid (v_i, v_k) \in \mathcal{E}^- \}$$
(10)

So, given a pair of nodes  $v_i$  and  $v_j$  in a directed signed social network,

$$C_s^I(v_i, v_j) = \{v_k | (v_k \in I_i^+ \land v_k \in I_j^+) \lor (v_k \in I_i^- \land v_k \in I_j^-)\}$$
(11)

$$C_{d}^{I}(v_{i}, v_{j}) = \{v_{k} | (v_{k} \in I_{i}^{+} \land v_{k} \in I_{j}^{-}) \lor (v_{k} \in I_{i}^{-} \land v_{k} \in I_{j}^{+})\}$$
(12)

$$C_{s}^{0}(v_{i}, v_{j}) = \{v_{k} | (v_{k} \in O_{i}^{+} \land v_{k} \in O_{j}^{+}) \lor (v_{k} \in O_{i}^{-} \land v_{k} \in O_{j}^{-})\}$$
(13)

$$C_{d}^{0}(v_{i}, v_{j}) = \{v_{k} | (v_{k} \in O_{i}^{+} \land v_{k} \in O_{j}^{-}) \lor (v_{k} \in O_{i}^{-} \land v_{k} \in O_{j}^{+}) \}$$
(14)

where  $C_s^I(v_i, v_j)$  means the common neighbors that hold the same evaluations on  $v_i$ and  $v_j$ ,  $C_d^I(v_i, v_j)$  means the common neighbors that hold different evaluations on  $v_i$  and  $v_j$ ,  $C_s^O(v_i, v_j)$  means the common neighbors that  $v_i$  and  $v_j$  hold the same evaluations on, and  $C_d^o(v_i, v_j)$  means the common neighbors that  $v_i$  and  $v_j$  hold different evaluations on. So, the similarity between  $v_i$  and  $v_j$  is:

$$sim(v_i, v_j) = \frac{|c_s^l(v_i, v_j)| - |c_d^l(v_i, v_j)| + |c_s^O(v_i, v_j)| - |c_d^O(v_i, v_j)|}{|N_i \cup N_j|}$$
(15)

where  $N_i = I_i \cup O_i$ ,  $N_j = I_j \cup O_j$ .

The basic similarity measure defined in directed signed social networks returns values into the interval [-1, 1] as well. Interval (0, 1] indicates two nodes are similar and interval [-1, 0) indicates two nodes are dissimilar. Specifically, 0 is a special value that denotes neither similar nor dissimilar. The larger the similarity value, the more similar they are.

#### 3.4 Propagation of Node Similarity

Based on the above discussion, the similarity values between all non-neighbor nodes in a graph G are zero. It is unreasonable. By propagating the similarity in the network, we can solve this problem. Notice that, we only propagate the similarity in the network in two hops according to the principle of balance theory [13, 14] that says "*the friend of my friend*", "*the enemy of my enemy is my friend*". That is to say, if two users have neither immediate common neighbors nor common related users (i.e. the similarity value determined by the basic similarity measure is not zero), the similarity between them is zero eventually. We define a extended similarity between two nodes  $v_i$  and  $v_j$ , denoted as  $exsim(v_i, v_j)$ .

$$exsim(v_i, v_j) = \begin{cases} sim(v_i, v_j), \text{ if } v_i, v_j \text{ have immediate common neighbors} \\ \frac{1}{|\mathcal{R}(v_i, v_j)|} \sum_{v_k \in \mathcal{R}} sim(v_i, v_k) \cdot sim(v_k, v_j), otherwise \end{cases}$$
(16)

where  $\mathcal{R}(v_i, v_j)$  is the set of common related users between user  $v_i$  and user  $v_j$ ,  $\mathcal{R}(v_i, v_j) = \{v_k \mid sim(v_i, v_k) \neq 0 \land sim(v_k, v_j) \neq 0\}.$ 

## 4 Experiments

In this section, we compare experimentally our approach with existing similarity measure algorithms.

#### 4.1 Data Sets

We use the Epinions [15] data set, which is a signed social network. We compute the in-degree and out-degree distributions of Epinions graph, treating both the positive and negative edges alike, as shown in Fig. 4.

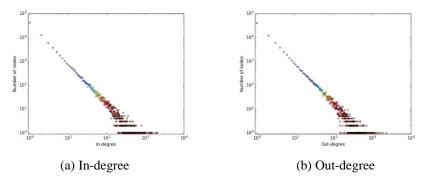


Fig. 4. Degree distributions of Epinions data set.

#### 4.2 Experimental Setup and Evaluation

To evaluate the proposed approach, we recommend a list of *k* friends (top-*k* list) to a target user. Our evaluation considers the division of neighbors of each target user into two sets: (1) the training set  $\mathcal{E}_T$  is treated as known information and, (2) the verification set  $\mathcal{E}_V$  is used for verification. We use *precision* and *accuracy* metric as performance measures for recommendations. In order to validate the effectiveness of our approach in signed social networks, we compare our model with FriendTNS algorithm and the Shortest Path algorithm, denoted as F-TNS and Shortest Path, respectively.

#### 4.3 Experimental results

In this section, we quantitatively compare our proposed method with baselines. Fig. 5 shows the results conducted in Epinions data set.

The horizontal axis of Fig.5 represents the number of friends that we recommend to a target user. From the Fig.5, we can see that our method can achieve better performance than baselines. The main reason is that our proposed method takes into account both the positive relationships and negative relationships. It shows from a side view that the negative links are important and valuable.

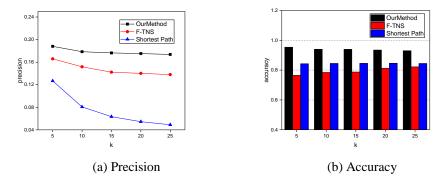


Fig. 5. Comparison of proposed method with baselines in Epinions.

## 5 Conclusion

In this paper, we study the similarity measure problem in signed social networks. We proposed a novel method to measure the similarity of nodes in signed social networks. Our method incorporates both positive and negative relationships simultaneously in signed networks. Extensive evaluations demonstrate the effectiveness of our approach.

Acknowledgement. This work is supported by NSF of China (No. 61602237), 973 Program (No. 2015CB352501), NSF of Shandong, China (No. ZR2013FQ009), the Science and Technology Development Plan of Shandong, China (No. 2014GGX101047, No. 2014GGX101019).

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